Effects of Suction on the Görtler Instability of Boundary Layers

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An analysis of the effects of suction on the Görtler instability of boundary layers is presented. Suction reduces the total growth of Görtler vortices and in this sense stabilizes the flow. The levels of suction that have to be used to achieve a significant stabilization are substantially higher than the level of suction sufficient to eliminate the Tollmien-Schlichting waves.

Nomenclature

A $= A^*/A_0^*$, ratio of the disturbance amplitude to the amplitude at the neutral point
G $= R(\delta_r/\mathcal{R})^{1/2}$, Görtler number
$\mathcal{R}$ = radius of curvature of wall
R $= U_\infty \delta_r/\nu = (U_\infty \tilde{\Phi}/\nu)^{1/2}$, Reynolds number
U, V = dimensionless basic-state velocities
U_∞ = freestream velocity, reference velocity
α = dimensionless wavenumber
β = dimensionless spatial amplification rate
δ_r $= (\nu\tilde{\Phi}/U_\infty)^{1/2}$, boundary-layer reference length
ϵ $= 1/R$, small parameter
Λ $= (U_\infty \lambda/\nu)(\lambda/\mathcal{R})^{1/2}$, dimensionless wavelength parameter
λ = dimensional vortex wavelength
ν = kinematic viscosity
Φ = streamwise coordinate
$\tilde{\Phi}$ = dimensional chordwise distance from leading edge
Φ_1 $= \epsilon\Phi$, slow scale
Ψ = normal-to-the-wall coordinate
θ = momentum thickness defined by Eq. (14)
$(\ )^*$ = dimensional quantity

Introduction

A LAMINAR-turbulent transition process begins when a laminar boundary layer loses its stability and evolves toward a turbulent form. A multitude of scenarios can result from the transition.[1] A boundary layer over a concave wall is subject to the action of centrifugal forces that under certain conditions, may destabilize the flow. The centrifugal instability leads to the generation of a new flow consisting of the laminar boundary layer with longitudinal counterrotating vortices superposed on it. Such a phenomenon is commonly referred to as the Görtler or the Taylor-Görtler instability.

Görtler instability considerably accelerates the transition process. Liepmann[2,3] noted that the transition in boundary layers above concave surfaces occurred much earlier than that which occurred above flat and convex surfaces. He concluded that the mechanism causing the transition on concave surfaces appeared to be different from that for flat and convex plates, and that the difference was due to the Görtler vortices. According to Wortmann[4-6] and Bippes,[7] different types of instabilities follow the onset of the centrifugal instability, but precede the bursts of turbulence. The exact sequence of events depends on the flow configuration and requires further studies. Nayfeh[8] showed that the spanwise periodicity of the mean flow, such as caused by Görtler vortices, accelerates the growth of the Tollmien-Schlichting waves and thus speeds up the transition process. In general, Görtler instability generates a new laminar flow that is less resistant to further instabilities and thus leads to an earlier transition. This is an undesirable consequence; therefore, there is an interest in analysis of the means to suppress the effects of the centrifugal instability. Suction has proved to be a very potent tool as far as Tollmien-Schlichting instability is concerned. Here, its usefulness in controlling the Görtler instability is evaluated.

Suction, which only slightly effects streamwise velocity components, considerably influences the normal-to-the-wall velocity components. Görtler[9] neglected effects of boundary-layer growth (parallel-flow approximation) and concluded that the stability characteristics were not influenced by the different forms of the velocity profiles, provided their momentum thickness did not differ considerably. In their study of Tollmien-Schlichting waves Saric and Nayfeh[10] showed that when a significant suction was introduced, the normal-to-the-wall velocity terms had to be retained. Kobayashi[11,12] studied the effects of suction on the Görtler stability in the case of asymptotic suction profile. This flow has the advantage of being a parallel boundary-layer type of velocity profile. His results showed the stabilizing influence of the suction but to a smaller extent then in the case of the Tollmien-Schlichting instability.[14] An analysis of the effects of the self-similar suction has been presented by Kobayashi[13] who used Görtler's equations[9] augmented with the normal-to-the-wall mean-flow velocity terms. The results showed the stabilizing influence of the suction, but again not to the extent expected for the Tollmien-Schlichting waves.[10] Recently Floryan and Saric[15] presented a model of the Görtler instability in which the effect of boundary-layer growth was incorporated in a rational way. They showed that the nonparallel terms entered the leading-order approximation of the disturbance equation, suggesting that suction may have a considerable effect on the stability characteristics. Moreover, they showed the proper ordering of the magnitudes of the disturbance velocity terms that reduced the problem to one of

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a single parameter. In this paper we use the instability model presented in Ref. 15 to assess the importance of suction.

Theory

The linear stability of an incompressible two-dimensional boundary layer is considered. The leading-order approximation for the disturbance equations described in Ref. 15 has the form of:

Mass

$$\beta u+\frac{dv}{d\Psi}+\alpha w=0 \tag{1}$$

Φ momentum

$$\beta Uu+u\frac{\partial U}{\partial \Phi_1}+\frac{\partial U}{\partial \Psi}v+V\frac{du}{d\Psi}=\frac{d^2u}{d\Psi^2}-\alpha^2 u \tag{2}$$

Ψ momentum

$$\beta Uv+\frac{\partial V}{\partial \Phi_1}u+\frac{\partial V}{\partial \Psi}v+V\frac{dv}{d\Psi}+2G^2Uu=-\frac{dp}{d\Psi}+\frac{d^2v}{d\Psi^2}-\alpha^2 v \tag{3}$$

z momentum

$$\beta Uw+V\frac{dw}{d\Psi}=\alpha p+\frac{d^2w}{d\Psi^2}-\alpha^2 w \tag{4}$$

In the above, Φ and Ψ denote coordinates generated from the streamlines of the inviscid flow over a curved surface, where u, v, and w are disturbance velocity components in the streamwise (Φ), normal-to-the-wall (Ψ), and spanwise (z) directions, respectively, p denotes pressure disturbance, U and V the streamwise and normal-to-the-wall basic-state velocity components, α the spanwise wavenumber, and β the spatial growth rate in the streamwise direction. The Görtler number $G=U_\infty\delta_r/\nu(\delta_r/\mathcal{R})^{1/2}$ is the critical stability parameter. The disturbance equations were made dimensionless by introducing a length scale $\delta_r=(\nu\tilde{\Phi}/U_\infty)^{1/2}$, freestream velocity U_∞ as a velocity scale for U, V, and u, and a "viscous velocity scale" ν/δ_r for v and w. This scaling, introduced by Floryan and Saric,[15] is crucial to the proper ordering of the disturbance velocity components. The coordinate Φ has been replaced by a slow scale $\Phi_1=\epsilon\Phi$ expressing a slow evolution of the flow in the streamwise direction. Here $\epsilon=\nu/U_\infty\delta_r\ll 1$, ν stands for the kinematic viscosity, and $\tilde{\Phi}$ denotes distance from the leading edge. Equations (1-4), supplemented by the standard boundary conditions of no slip and no penetration at the wall and decay of the disturbances far away from the wall,

$$u=v=w=0 \quad \text{at} \quad \Psi=0,\infty \tag{5}$$

form an eigenvalue problem for the parameters (α,β,G). For the case with suction, the disturbances would not necessarily vanish completely at the surface. The boundary conditions [Eq. (5)] are, however, approximately valid when the holes or suction slots are fine enough.[12] The relaxation of the no-penetration condition was studied by Gaponov[16] for calculations of the critical Reynolds number of Tollmien-Schlichting waves. However, Lekoudis[17] showed that this may not be an important consideration.

The eigenproblem [Eqs. (1-5)] has to be solved numerically. The appropriate numerical procedure is described in Ref. 15.

Asymptotic Suction

The investigation of the effects of suction for the asymptotic suction profile was originated by Kobayashi.[11,12] Equations (1-4) reduce to his equations as given in Ref. 11 for the asymptotic suction case. The asymptotic suction profile has the form of

$$U=1-\exp(-\gamma\Psi) \tag{6}$$

where Ψ is normalized with δ_r and the suction parameter γ is defined as

$$\gamma=(v_0/U_\infty)R \tag{7}$$

Here $v_0=$ const denotes dimensional velocity at the wall and $R=U_\infty\delta_r/\nu$. The neutral stability curves for different values of suction parameter are given in Fig. 1. For wavenumbers of practical importance[18] the suction stabilizes the flow as expected. However, these changes are not as dramatic as the changes of the critical Reynolds number for the Tollmien-Schlichting waves.[14] It appears that suction slightly destabilizes the flow for wavenumbers greater than unity. This destabilization has no practical value, but suggests that a global balance of forces rather than local conditions governs stability.

Self-Similar Suction

The investigation of the effects of the self-similar suction profile was originated by Kobayashi[13] who employed Görtler's equations[9] augmented with the normal-to-the-wall mean flow velocity terms. The resulting equations contained only a few of the non-parallel terms that were shown to be important in the leading-order approximation[15] and have been included in Eqs. (1-4). The mean flow velocity components in the self-similar suction case have the following form[10]

$$U^*/U_\infty=U(\Phi_1,\Psi), \qquad V^*/U_\infty=\epsilon V(\Phi_1,\Psi) \tag{8}$$

They are expressed by means of a stream function $\mathcal{H}$ as

$$U=\frac{\partial\mathcal{H}}{\partial\Psi}, \qquad V=-\frac{\partial\mathcal{H}}{\partial\Phi_1} \tag{9}$$

The independent variables are transformed from the (Φ_1,Ψ) plane to the (η,ξ) plane by the following transformation:

$$\xi=\sqrt{\Phi_1}, \qquad \eta=\Psi/\xi \tag{10}$$

Substituting Eqs. (8-10) into the boundary-layer equations and defining the streamfunction $\mathcal{H}(\xi,\eta)$ as

$$\mathcal{H}(\xi,\eta)=\xi f(\eta)-2\int\gamma d\xi \tag{11}$$

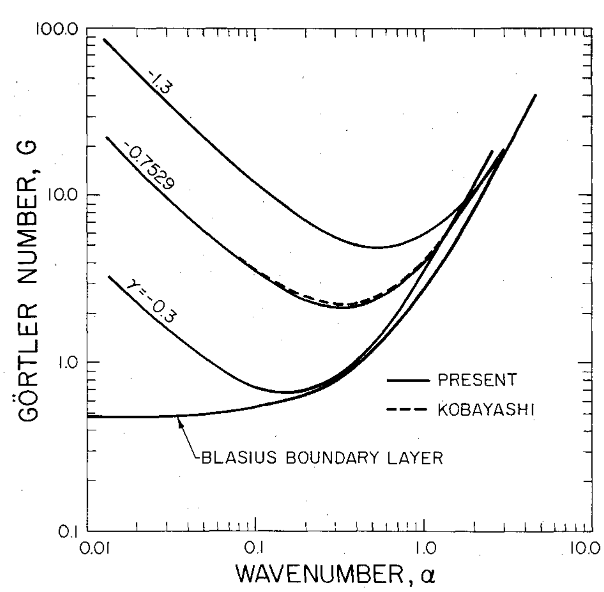

Fig. 1 Neutral stability curves for the asymptotic suction velocity profile, comparison of the present theory with Kobayashi.[11]

where

$$\gamma = \xi v_0(\xi) \quad (12)$$

one obtains the following differential equation describing the mean flow

$$f_{\eta\eta\eta} + \frac{1}{2} f f_{\eta\eta} - \gamma f_{\eta\eta} = 0 \quad (13)$$

where subscript η refers to $d/d\eta$ and v_0 denotes velocity at the wall. For the self-similar suction the suction coefficient γ = const and v_0 = const/$\xi \simeq \Phi^{-1/2}$ [the reader should note that $v_0^* = U_\infty (U_\infty \bar{\Phi}/\nu)^{1/2}$]. Here $\gamma = 0(1)$ and the dimensional suction velocity is $0(\epsilon)$ to retain the boundary-layer approximation. Note that when δ_r is evaluated for a particular chord position, then $\xi = 1$ and $\Psi = \eta$. The momentum thickness is given as

$$\theta/\delta_r = 2[f''(0) + \gamma] \quad (14)$$

Values of $f''(0)$ are given in Table 1 in order to assist the reader who may be interested in expressing results in terms of θ rather than δ_r.

Figures 2 and 3 display the effects of the self-similar suction on the stability characteristics. A small suction ($\gamma < -0.5$) reduces the critical Görtler number; it seems to destabilize the boundary layer for wavenumbers $\alpha < 0.2$, and slightly stabilize it for wavenumbers $0.2 < \alpha < 2.0$. It appears that the boundary layer with suction given by $\gamma \approx -0.5$ is the most unstable as far as the critical Görtler number is concerned. If the suction is increased above this level, the critical Görtler number noticeably increases. The variation of the critical Görtler number with the increase of suction remains qualitatively the same when the data are scaled with the momentum thickness θ, as defined by Eq. (14), rather than length scale δ_r. The level of suction required for a noticeable increase of the critical Görtler number is by an order of magnitude higher than the suction required to stabilize Tollmien-Schlichting waves.[10] In fact, at suction levels $\gamma < -0.5$, Tollmien-Schlichting waves are practically eliminated and Görtler vortices may dominate the transition process. Variations of the critical Görtler number may be correlated with the suction parameter γ, with the help of Table 1 and Figs. 2 and 3. The minimum value of the critical Görtler number corresponds to the disappearance of the normal-velocity component at the edge of the boundary layer as seen in Table 1. This is a puzzling phenomenon and is not well understood yet.

Table 1 Transverse velocity component at the edge of the boundary layer for the different values of the suction parameter γ

γ	V_{edge}	$f''(0)$
0.0	0.8604	0.3321
−0.1	0.6657	0.4061
−0.2	0.4883	0.4833
−0.3	0.3240	0.563
−0.4	0.1697	0.645
−0.5	0.0233	0.7287
−0.51	0.0091	0.7374
−0.5164	0.0000	0.7428
−0.52	−0.0051	0.7459
−0.53	−0.0193	0.7544
−0.54	−0.0334	0.7629
−0.6	−0.1167	0.8144
−0.7	−0.2516	0.9014
−0.8	−0.3822	0.9897
−1.0	−0.6333	1.17
−1.2	−0.8742	1.353

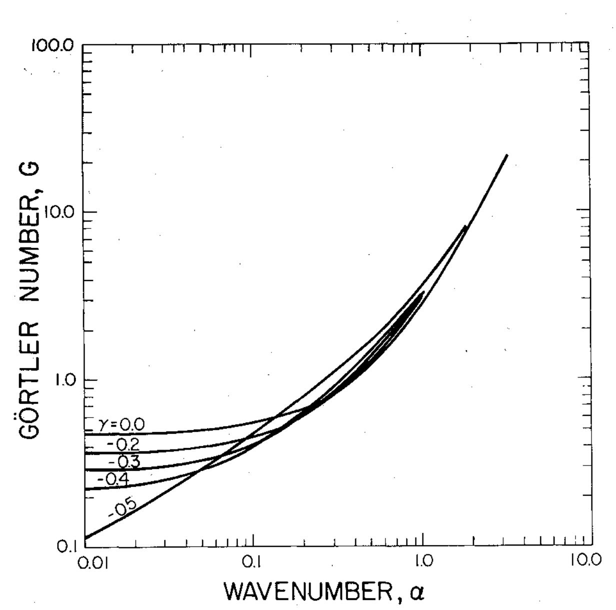

Fig. 2 Neutral stability curves for the Blasius boundary layer with self-similar suction.

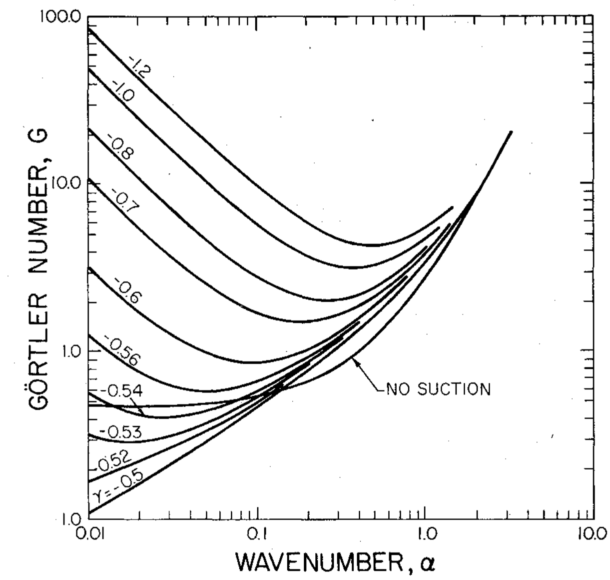

Fig. 3 Neutral stability curves for the Blasius boundary layer with self-similar suction.

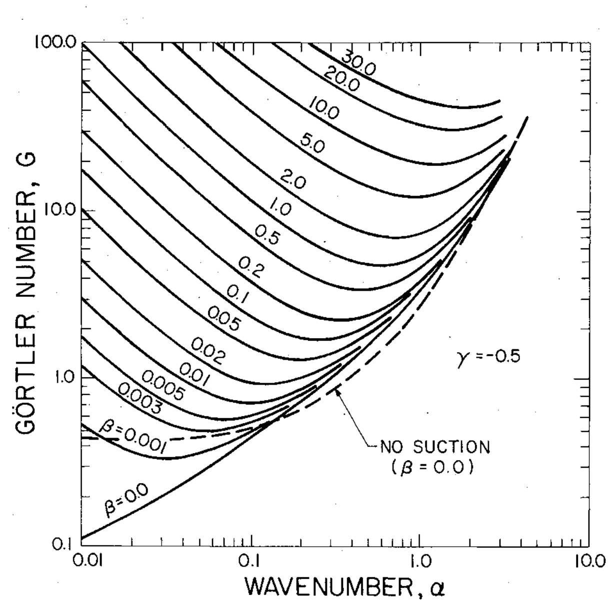

Fig. 4 Curves of constant amplification rate as a function of Görtler number G and wavenumber α for the Blasius boundary layer with self-similar suction of $\gamma = -0.5$.

Curves of constant amplification rates for two different levels of suction are given in Figs. 4 and 5. An increased suction compresses curves of small amplifiction rates and moves them toward higher Görtler numbers. Curves for amplifications rates $\beta = 10 \div 30$ are practically not influenced by suction. Since the growth rates rather than the critical stability conditions play a crucial role for the Görtler instability,[18] the influence of suction should be judged on the basis of the total growth of the vortices. The total growth may be computed according to the formula

$$A = \exp\left[\int_{\Phi_0^*}^{\Phi^*} \beta^* \mathrm{d}\Phi^*\right] = \exp\left[\int_{G_0}^{G} \frac{4}{3}\frac{\beta}{G}\mathrm{d}G\right] \qquad (15)$$

where $A = A^*/A_0^*$ and $A(G_0) = 1$. Here A denotes amplitude of the disturbances and subscript 0 the initial conditions. Each integration begins at the neutral curve and follows the same vortex defined by the constant dimensional wavelength λ downstream. The results are presented in Figs. 6-8 in terms of the wavelength parameter $\Lambda = U_\infty \lambda/\nu (\lambda/\mathcal{R})^{1/2}$. The use of this parameter has a distinct advantage over the use of wavenumber parameter α because α varies with streamwise location.[18]

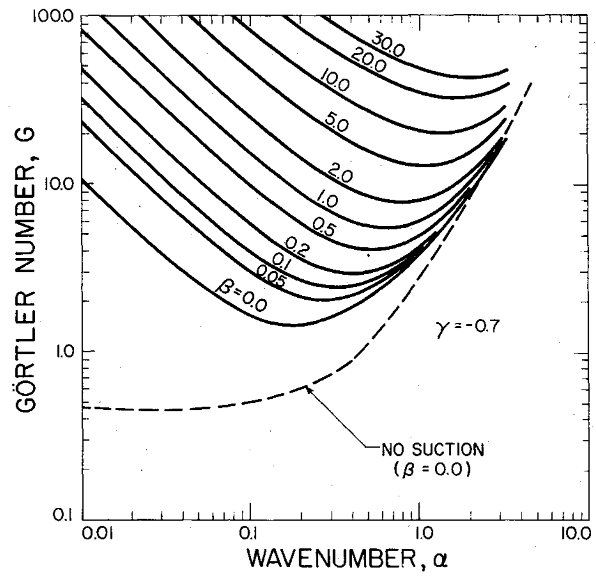

Fig. 5 Curves of constant amplification rate as a function of Görtler number G and wavenumber α for the Blasius boundary layer with self-similar suction of $\gamma = -0.7$.

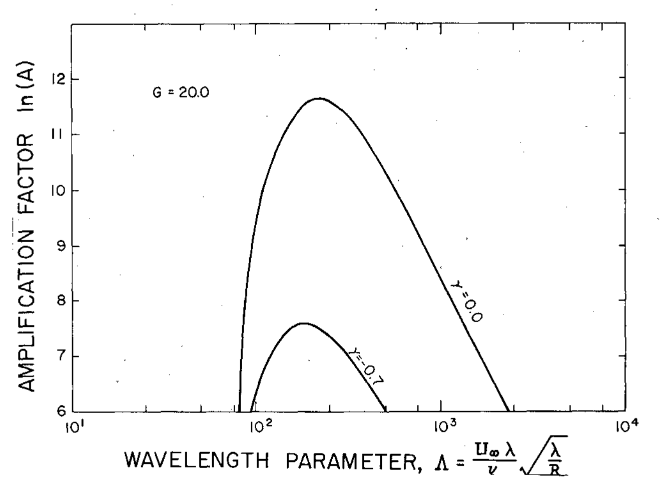

Fig. 6 Curves of total growth of disturbances as a function of the wavelength parameter Λ at the streamwise location corresponding to Görtler number $G = 20.0$ (calculations are for the Blasius boundary layers without suction and with self-similar suction of $\gamma = -0.7$).

The most amplified disturbances correspond to $\Lambda \simeq 210$ for the no-suction case.[18] Figure 6 displays the total amplification of disturbances that occurred between the neutral curve and a chordwise location corresponding to Görtler number $G = 20.0$. The results show that suction reduces the total growth of the disturbances and slightly decreases the wavelength parameter corresponding to the most amplified vortices.

Figure 7 illustrates variations of the total growth of disturbances as a function of the Görtler number for three values of wavelength parameter corresponding to vortices observed experimentally.[18] Suction reduces the total growth; however, the growth process is qualitatively unaffected.

We may conclude, by comparing Figs. 2 and 3 with Fig. 8, that also suction corresponding to $\gamma = -0.5$ seems to locally destabilize the boundary layer in terms of the critical Görtler number, it reduces the total streamwise growth, and in this sense stabilizes the boundary layer. Let us examine Fig. 8 and use Liepmann's[2,3,18] value of the transitional Görtler number $G = 16.6$, which corresponds to the total growth of $A = \exp(8.93)$ of the most amplified disturbances[18] ($\Lambda = 210$) in a boundary layer without suction. A suction of $\gamma = -0.14$ is required to keep these disturbances at the same level of amplification at $G = 17.6$. At $G = 18.6$ a suction of $\gamma = -0.28$

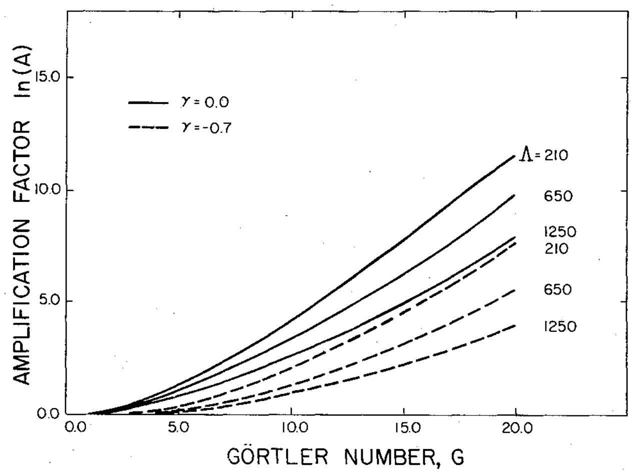

Fig. 7 Curves of total growth of disturbances as a function of Görtler number for different values of wavelength parameter Λ (calculations are for the Blasius boundary layers without suction and with self-similar suction of $\gamma = -0.7$).

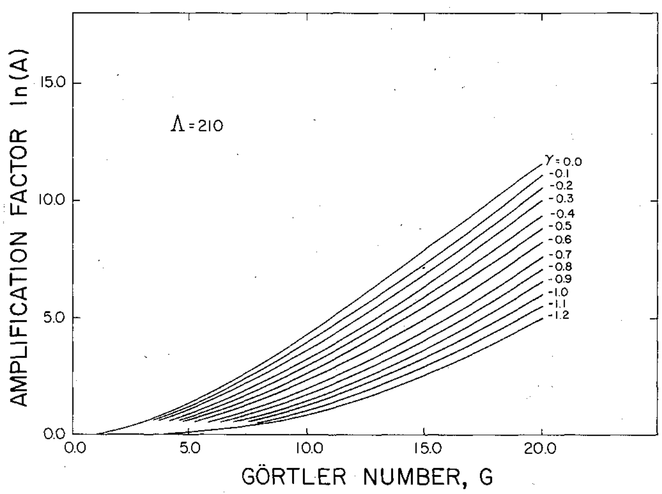

Fig. 8 Curves of total growth of disturbances as a function of Görtler number for the Blasius boundary layer with different levels of self-similar suction (the value of the wavelength parameter Λ corresponds to the disturbances of the maximum total growth for the no-suction case).

is required at $G = 19.6$ a suction of $\gamma = -0.42$ must be applied. Thus, suction is not effective in significantly delaying the transition caused by Görtler vortices. By comparison, a suction of $\gamma = -0.2$ practically eliminates Tollmien-Schlichting waves.

Conclusions

The effect of suction on the Görtler instability of boundary layers have been studied for the cases of asymptotic suction and self-similar suction. The present theory reduces to the formulation presented by Kobayashi[11] in the case of an asymptotic suction profile. Numerical results agree with those of Ref. 11 and show that the asymptotic suction profile is stabilized by the increased suction. The increase of suction up to $\gamma = -0.5$ reduces the critical Görtler number in the self-similar case. A further increase of suction increases the critical Görtler number. The total growth of disturbances is reduced regardless of the level of suction and, therefore, it may be concluded that suction stabilizes the flow in the self-similar case too. Suction of considerable higher levels than in the case of Tollmien-Schlichting waves is required to achieve a meaningful stabilization.

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